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A NEW OPTIMAL SELECTION OF A RIDGE REGRESSION PARAMETER

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ABSTRACT

In this paper, a new ridge regression parameter is proposed for the ridge regression estimator. The idea of this proposal is based on combining the best well known ridge parameter estimators and sample size. The superiority of the new biasing parameter estimator is investigated through the mean squared error criterion (MSE) criterion, and the relative efficiency criterion. Therefore, simulation experiments are conducted and found that the new ridge estimator has less bias, and smallest MSE, in all degrees of multicollinearity, in all levels of error variances, and for all sample sizes. Also, it is found that the length of the new proposed estimator converges to the length of the true regression parameter. The results of the new ridge estimator are compared to the ordinary least squares (OLS) estimator and all the ridge estimators considered in the new estimator and some other recent ridge estimators. It can be concluded that our new estimator has a little bias and it is a consistent estimator than other ridge estimators considered in this paper and the OLS estimator.

KEYWORDS: Bias, Consistency, Mean Squared Error (MSE), Multicollinearity, Ordinary Least Squares (OLS), Ridge Parameters, Ridge Estimators

1. INTRODUCTION

Many regression procedures have been proposed to deal with challenging data problems such as multicollinearity. Ridge regression, proposed by Hoerl and Kennard (1970), is one of the most widely used estimation technique to overcome the problem of multicollinearity. Ridge regression can be shown as a modification of least squares method. This modification is based on introducing a biasing factor to shrink the length of the least squares estimator (OLS).

Consider the following multiple linear regression model:

$$Y = X\beta + \varepsilon \,, \tag{1.1}$$

where Y is an (n×n) column vector of the dependent variable, X is an (n×p) matrix of regressors, β is a (p×1) vector of unknown parameters to be estimated, and ε is an (n×1) vector of errors distributed as $N(0,\sigma^2 I_n)$. the least squares estimator of β is given by:

$$\hat{\beta} = (X'X)^{-1} X'Y \,. \tag{1.2}$$

Ridge regression strategy is based on adding a small quantity to the diagonal elements of $X'X$. (see Hoerl and Kennard, 1970a; 1970b).

Thus the ridge estimator is defined as follows:

$$\hat{\beta}_k = \left(X'X + kI\right)^{-1} X'Y \ . \tag{1.3}$$

This estimator is called the ordinary ridge estimator (ORE), where k is a small positive quantity which is greater than zero called the ridge parameter. The generalized form of (1.3) is introduced by Hoerl and Kennard (1970) as follows:

$$\hat{\beta} = \left(X'X + K\right)^{-1} X'Y \tag{1.4}$$

Where K is a diagonal matrix, with diagonal elements $k_i>0$ for i=1,2,…,p. Eq.(1.4) is defined as a generalized ridge estimator (GRE). When k_i =0, ordinary least squares estimator (OLS) is obtained, and when k_i=k for all i=1,2,…,p, ordinary ridge estimator (ORE) is obtained.

The ridge estimator is biased but it has smaller mean squared error (MSE) than OLS estimator

After Horel and Kennard (1970), the area of ridge regression is studied by many authors who introduced different ridge parameters. Some of them are Hoerl et al (1975); McDonald and Galarneau (1975); Lawless and Wang (1976); Hocking et al. (1976); Numura (1988), Saleh and Kibria (1993); Kibria (2003); Khalaf and Shukur (2005); Alkhamisi et al. (2006); Adnan et al. (2006); Yan (2008); Muniz and Kibria (2009); Yan and Zhao (2009); Al-hassan (2010); Manson et al. (2010); Muniz et al. (2012); Asar et al. (2014); Dorugade (2014).

From the previous studies on ridge regression, it is found that there is no definite rule in estimating the ridge parameter. All the introduced rules are depended on the three mentioned rules of Horel and Kennard (1970), Hoerl et al (1975), and Lawless and Wang (1976).

All the contributions in ridge parameters are just adjustments of the ridge parameters introduced by Horel and Kennard (1970), Hoerl et al (1975), and Lawless and Wang (1976). Therefore, in this paper, a new ridge parameter is obtained which is depended mainly on the combining the three previous rules which gave best results in most studies and the idea of sample size. The idea of sample size is neglected in all studies when estimating the ridge parameter. Therefore, in this work, the sample size will be a main factor in estimating the ridge parameters. The performance of the resulting ridge estimator will be measured using some criteria such as mean squared error (MSE).

This paper is organized as follows. Section (2) introduces different methods of estimating the ridge parameters. The proposed new ridge parameter and the new ridge estimator are presented in section (3). The Mont Carlo simulation study, real-life data, and the performance criteria of the new estimator are given in section (4). Conclusions and discussions are considered in section (5).

2. ESTIMATING THE RIDGE PARAMETERS

Consider the eigen decomposition of the matrix $X'X$ as:

$$X'X = Q\Lambda Q' ,$$

where Q is the eigenvectors of $X'X$ and Λ is a diagonal matrix of the eigenvalues of $X'X$.

The canonical form of model (1.1) can be written as:

$$Y = Z\alpha + \varepsilon , \tag{2.1}$$

where Z= XQ, and $\alpha = Q'\beta$. The OLS estimator of α is:

$$\hat{\alpha} = \Lambda^{-1} Z'Y = Q'\hat{\beta}. \tag{2.2}$$

The ordinary ridge estimator of α is defined as:

$$\hat{\alpha}_k = (\Lambda + kI)^{-1} Z'Y, \tag{2.3}$$

Which corresponds to $\hat{\beta}$ in (1.3). The generalized ridge estimator of α is:

$$\hat{\alpha}_K = (\Lambda + K)^{-1} Z'Y, \tag{2.4}$$

Which correspond to $\hat{\beta}_K$ in (1.4), K=diag(k_1, k_2,…, k_p) and k_i>0.

Horel and Kennard (1970) showed that the value of k_i which minimizes the MSE of k_i is defined as:

$$k_i = \frac{\sigma^2}{\alpha_i^2}, \tag{2.5}$$

Where, σ^2 is the error variance of the model (1.1), and α_i is the ith element of α.

There is no definite rule for estimating the ridge parameter. However, several techniques have been proposed, most of them are transformations or adjustments of the three methods introduced by Hoerl and Kennard (1970a,b), Hoerl et al. (1975), and Lawless and Wang (1976). These parameters gave best results in most researches and will be presented here, with other ridge parameter.

2.1 Hoerl and Kennard (1970)

Hoerl and Kennard (1970a,b) proposed estimating the ridge parameter denoted by $\hat{k}_1$ as follows:

$$\hat{k}_1 = \frac{\hat{\sigma}^2}{\hat{\alpha}_{\max}^2} \tag{2.6}$$

Where $\hat{\sigma}^2 = \frac{\sum (y - \hat{y})^2}{n - p}$ is the estimated residual mean square, $\hat{\alpha}_i$ is the ith element of the vector $\hat{\alpha} = Q'\hat{\beta}$, and Q is an orthogonal matrix, and $\hat{\alpha}_{\max}^2$ is the maximum value of $\hat{\alpha}_i^2$.

2.2 Hoerl, Kennard, and Baldwin (1975)

Hoerl, Kennard, and Baldwin (1975) proposed estimating the ridge parameter using the harmonic mean of $\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ denoted by $\hat{k}_2$ as follows:

$$\hat{k}_2 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\alpha}_i^2} \quad (2.7)$$

2.3 Lawless and Wang (1976)

Lawless and Wang (1976) proposed estimating the ridge parameter using the harmonic mean of $\frac{\hat{\sigma}^2}{\lambda_i\hat{\alpha}_i^2}$ denoted by $\hat{k}_3$ as follows:

$$\hat{k}_3 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2}. \quad (2.8)$$

All the three estimated ridge parameters are based on the least squares estimators of both σ^2 and α, defined as follows:

$$\hat{\sigma}^2 = \frac{(y - z\hat{\alpha})'(y - z\hat{\alpha})}{n - p}$$

and

$$\hat{\alpha} = \Lambda^{-1}z'y. \quad (2.9)$$

There are many contributions in the area of selecting the ridge parameter; all of them are adjusted versions of the three previous methods.

2.4 Other Suggested Ridge Estimators

Recently, Kibria (2003) proposed some new ridge parameters applied to the generalized ridge regression approach. He used the geometric mean of (2.5) denoted by $\hat{k}_4$ as follows:

$$\hat{k}_4 = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p}\hat{\alpha}_i^2\right)^{\frac{1}{p}}}, \quad (2.10)$$

Also, he used the median of (2.5) for p≥3 denoted by $\hat{k}_5$ as follows:

$$\hat{k}_5 = median\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right), \qquad i=1,2,\ldots,p \quad (2.11)$$

Khalaf and shukur (2005) investigated a new estimate of the ridge parameter k as adjusted version of (2.6) denoted by $\hat{k}_6$ as follows:

$$\hat{k}_6 = \frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}^2_{max}}, \tag{2.12}$$

Where λ_{max} is the maximum eigenvalue of the matrix $X'X$.

Alkhamisi et al. (2006) suggested some ridge estimators based on, the ridge parameter estimate introduced by Khalaf and shukur (2005) in (2.12), respectively, as the mean, maximum value, and median of $\hat{k}_6$ denoted by $\hat{k}_7$, $\hat{k}_8$, and by $\hat{k}_9$:

$$\hat{k}_7 = mean(\hat{k}_6) = \frac{1}{p}\sum_{i=1}^{p}\left(\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2}\right), \tag{2.13}$$

$$\hat{k}_8 = \max(\hat{k}_6) = \max\left(\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2}\right), \text{ i=1,2,…,p,} \tag{2.14}$$

$$\hat{k}_9 = median(\hat{k}_6) = median\left(\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_i\hat{\alpha}_i^2}\right), \text{ i=1,2,…,p.} \tag{2.15}$$

Yan (2008); Muniz and Kibria (2009); Yan and Zhao (2009); Al-hassan (2010); Manson et al. (2010); Muniz et al. (2012); Asar et al. (2014); Dorugade (2014); Adnan et al. (2016), and others introduced some ridge parameters which are adjusted versions of the previous ones presented here.

3. THE PROPOSED RIDGE ESTIMATOR

3.1 The Methodology

It is found that all the proposed ridge parameters are different adjustments of the above three ridge parameters introduced by Hoerl and Kennard (1970), Hoerl et al.(1975), and Lawless and Wang (1976), introduced in (2.5), (2.6), and (2.7), respectively. Also, all these adjustments did not consider the sample size in the formulas of the ridge parameters.

Particularly, ridge regression performs well if there is a subset of true coefficients which has small length. But when all of the true coefficients have large length, ridge regression doesn't do well. However, using small values of k, it can still outperform the least squares estimator. (See New House and Oman, 1970).

Choosing an appropriate value of k is a difficult and an important task. Hoerl and Kennard (1970) showed that the value of k is affected by the length of the true regression coefficients as follows:

1. If the length of the true regression coefficients is large, the range of best values of k will be smaller, 0<k<1.

2. If the length of the true regression coefficients is small, the range of best values of k will be larger, k>0.

3.2 The Proposed Ridge Parameter

The proposed ridge parameter is a function of the three parameters' formulas introduced in (2.5), (2.6), (2.7), and take into consideration the sample size n as follows:

$$\hat{k}_{new} = \frac{np\hat{\sigma}^2}{\left[p\hat{\alpha}_{\max}^2 \times \sum_{i=1}^{p}\hat{\alpha}_i^2 \times \sum_{i=1}^{p}\lambda_i\hat{\alpha}_i^2\right]^{1/p}}. \quad (3.1)$$

The above proposed $\hat{k}_{new}$ will have different positive values according to the different scenarios presented in the simulation study.

3.3 Evaluation of the New Ridge Estimator

The mean squared error (MSE) and the relative efficiency criteria are used as measures of the quality of the estimator. So, the MSE is used as a criterion to evaluate the new ridge estimator and the other ridge estimators compared to the ordinary least squares (OLS) estimator. The relative efficiencies are computed to compare the proposed estimator to the OLS estimator and to each other ridge estimator presented here.

The MSE of the ridge estimator in (2.4) is defined as:

$$MSE(\hat{\alpha}_k) = Var(\hat{\alpha}_k) + [bias(\hat{\alpha}_k)]^2 = \sigma^2\sum_{i=1}^{p}\frac{\lambda_i}{(\lambda_i + k)^2} + k^2\sum_{i=1}^{p}\frac{\alpha_i^2}{(\lambda_i + k)^2}, \quad (3.2)$$

and the MSE of OLS estimator in (2.9) is defined as:

$$MSE(\hat{\alpha}) = Var(\hat{\alpha}) = \sigma^2\sum_{i=1}^{p}\frac{1}{\lambda_i}, \quad (3.3)$$

Theorem (1): Hoerl and Kennard (1970) Exitence Theorem (4.3)

There always exists a k>0 such that: $MSE(\hat{\alpha}_k) < MSE(\hat{\alpha})$.

Proof:

By taking the first derivative of each term in (3.2) w.r.t. k, the following equation will be obtained:

$$\begin{aligned} dMSE(\hat{\alpha}_k)/dk &= dVar(\hat{\alpha}_k)/dk + d[bias(\hat{\alpha}_k)]^2/dk \\ &= -2\sigma^2\sum_{i=1}^{p}\frac{\lambda_i}{(\lambda_i + k)^3} + 2k\sum_{i=1}^{p}\frac{\lambda_i\alpha_i^2}{(\lambda_i + k)^3} \end{aligned} \quad (3.4)$$

By equating Eq.(3.4) with zero, the ridge parameter k that satisfy the condition $MSE(\hat{\alpha}_k) < MSE(\hat{\alpha})$ will be :

$$k = \frac{\sigma^2}{\sum_{i=1}^{p}\alpha_i^2}, \quad (3.5)$$

The above condition could be written as follows:

$$MSE(\hat{\alpha}) - MSE(\hat{\alpha}_k) = \sigma^2 \sum_{i=1}^{n} \frac{1}{\lambda_i} - \sigma^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2} > 0 \quad \text{if } k > 0$$
$$= 0 \qquad \text{if } k = 0$$

The relative efficiency of the estimator is defined as the ratio of the MSE of the new and other ridge estimator to the MSE of the OLS estimator defined in (3.3).

4. SIMULATION EXPERIMENTS

In this work, the properties of the proposed estimator and other estimators are examined through Monte Carlo simulations. The properties of the different estimators are compared using the average mean square error (MSE) criterion, and the relative efficiency (RE) criterion of the new and different ridge estimators with respect to the OLS estimator and to each other ridge estimator presented here.

The estimators used in this study are 11 estimators abbreviated as follows:

E_0: The OLS estimator in (2.2).

E_1: The ridge estimator of Hoerl and Kennard (1970), based on the ridge parameter in (2.6).

E_2: The ridge estimator of Hoerl, Kennard, and Baldwin (1975), based on the ridge parameter in (2.7).

E_3: The ridge estimator of Lawless and Wang (1976), based on the ridge parameter in (2.8).

E_4: The ridge estimator of Kibria (2003), based on the ridge parameter in (2.10).

E_5: The ridge estimator of Kibria (2003), based on the ridge parameter in (2.11).

E_6: The ridge estimator of Khalaf and shukur (2005), based on the ridge parameter in (2.12).

E_7: The ridge estimator of Alkhamisi et al. (2006), based on the ridge parameter in (2.13).

E_8: The ridge estimator of Alkhamisi et al. (2006), based on the ridge parameter in (2.14).

E_9: The ridge estimator of Alkhamisi et al. (2006), based on the ridge parameter in (2.15).

E_{new}: The proposed ridge estimator, based on the ridge parameter in (3.1).

The design of the simulation studies are based on the following elements:

1. The Model Used for Generating Data:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_p X_{ip} + \varepsilon_i, \qquad i=1,2,\ldots,n,\ p=4,\ 10, \qquad (4.1)$$

where $\varepsilon_i \sim N(0, \sigma^2)$, and without loss of generality β_0 is taken to be zero. Series values of σ^2 are used which include 0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0 to study the effect of the error term on the performance of the estimators. Also, different levels of collinearity among regressors are used.

2. The Procedures Used For Generating the Regressors and the Response Variables

Following McDonald and Galarneau (1975), regressors generated by:

$$X_{ij} = \left(1-\rho^2\right)^{\frac{1}{2}} Z_{ij} + \rho Z_{ip}, \qquad i=1,2,\ldots,n, \qquad j=1,2,\ldots,p, \tag{4.2}$$

where ρ represents the correlation between regressors. Different values of ρ are used such as: 0.5, 0.7, 0.9, 0.99, and 0.999, to study the effect of the different estimators and the proposed one. Z_{ij}*'s* are independent pseudo random numbers distributed as standard normal distribution with mean zero and unit variance. The response variable is generated as seen in (4.1).

3. Choosing the True Parameter Vector

- The vector β is chosen to have a length equal one, so the vector β is selected to be the eigenvector corresponding to the largest eigenvalue of the matrix $X'X$, since the MSE will be minimized when using this vector. It is found that when $\beta = V_1$ (the eigenvector corresponding to the largest eigenvalue) the MSE is minimized, but when $\beta = V_p$ (the eigenvector corresponding to the smallest eigenvalue) the MSE is maximized. (See Newhouse and Oman, 1971).
- The vector β is chosen to have a length greater than one. For p=4, β is selected to be β=(1, 1, 1, 1), and for p=10, β is selected to be β=(1, 1, 1, 1, 1, 1, 1, 1, 1, 1).

4. Sample Sizes

Different sample sizes are used to study the effect of the new proposed ridge estimator and the other estimators, n=20, 50, 100, 200, and 500 observations, which cover all models with different number of regressors, different degree of multicollinearity and different values of σ^2.

For different values of n, p, ρ, β and σ^2, different models are generated. For every model the experiment are repeated 5000 times, the average MSE, and the average relative efficiency are computed for the proposed estimator and the other 9 ridge estimators.

5. SIMULATION RESULTS

5.1 When the Length of the True Regression Coefficient=1, And P=4

Four different values of correlation coefficient are used (ρ=0.7, 0.9, 0.99, 0.999), five different sample sizes (n=20, 50, 100, 200, 500), and eight different values of σ^2 (σ^2=0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0). The total experiments = 4×5×8=160 experiments for p=4.

The results of the experiments are shown in Tables (1:20) in Appendix (A). It is found that the average mean squared error (AMSE) of the new proposed estimator (E_{new}) is less than all the ridge estimators and the OLS estimator in 156 experiments for all sample sizes, all levels of σ^2, and all levels of correlation coefficient ρ. In four experiments, E_5 and E_4 are combated our new estimator E_{new}. When n=20, σ^2=1.0, 2.0, and ρ=0.7 E_5 gave AMSE less than E_{new}. Also, when n=20, σ^2=0.5, 1.0, and ρ=0.999 E_4 gave AMSE less than E_{new} .The relative efficiency (RE) between each estimator and the

OLS estimator is presented in Tables (5-1:5-4) , and Figure (5-1). In Figures (5-1-a), (5-1-b), (5-1-c) and (5-1-d), it is clear that E_{new} is the best estimator in all cases except in Figure(5-1-a) when n=20, σ^2=1.0, 2.0, and ρ=0.7 where E_5 is more efficient than all the estimators. Also, in Figure (5-1-d) when n=20, σ^2=0.5, 1.0, and ρ=0.999 where E_4 is more efficient than all the estimators. Tables (5-1:5-4) shows that the efficiency of the new ridge estimator, E_{new}, will be greater for larger values of σ^2 compared to other ridge estimators. These results indicate that the new ridge estimator, E_{new}, is preferred when σ^2 is greater than one. Table (5-5) and Figure (5-2) show the true and estimated Regression parameters length for the OLS estimator, ridge estimators, and the new ridge estimator when p=4, ρ=0.9, σ^2=5, and sample sizes n=20, 50, 100, and 200. It is found that as n increase the length of all the estimators decrease, but the new estimator, E_{new}, is the only estimator which its length approaches to the length of the true regression parameter. Also, Table (5-6) shows that the new ridge estimator, E_{new}, has less absolute value of bias compared to other ridge estimator when p=4, ρ=0.9, and σ^2=5. These results assure that the new estimator, E_{new}, is a less biased, has less MSE and its length converges to the true parameter length, so it is a consistent estimator of β.

Table 5 1: The Relative Efficiencies of the New Ridge Estimator and Other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, and ρ=0.7

σ^2 / Estimator	0.1	0.5	1	2	5	10	15	20
E_0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
E_1	1.002	1.018	1.039	1.066	1.196	1.114	1.120	1.123
E_2	1.002	1.039	1.112	1.227	1.368	1.355	1.363	1.367
E_3	1.000	1.001	1.001	1.003	1.005	1.004	1.005	1.005
E_4	1.002	1.054	1.313	5.317	3.736	1.693	1.698	1.703
E_5	1.002	1.067	***1.781***	***9.933***	1.354	2.068	1.919	1.856
E_6	1.002	1.124	1.776	2.338	1.230	1.467	1.427	1.409
E_7	1.002	1.048	1.212	1.355	1.117	1.169	1.160	1.156
E_8	1.002	1.093	1.725	2.338	1.230	1.467	1.427	1.409
E_9	1.002	1.041	1.070	1.082	1.098	1.090	1.091	1.092
E_{new}	***1.012***	***1.241***	*1.734*	2.801	***6.626***	***9.089***	***11.996***	***14.614***

Note that: bold and italic values indicate the most efficient estimators (larger values)

Table (5 2): The Relative Efficiencies of the New Ridge Estimator and Other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, And ρ=0.9

σ^2 / Estimator	0.1	0.5	1	2	5	10	15	20
E_0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
E_1	1.015	1.242	1.528	1.337	1.627	1.566	1.547	1.539
E_2	1.023	1.474	2.049	2.317	2.606	2.617	2.617	2.615
E_3	1.000	1.006	1.013	1.015	1.018	1.018	1.018	1.018
E_4	1.025	2.427	2.308	7.598	7.267	27.211	26.783	17.512
E_5	1.025	1.505	2.377	11.190	3.356	3.959	4.215	4.355
E_6	1.042	1.232	1.486	4.635	2.158	2.329	2.393	2.428

E_7	1.022	1.175	1.227	1.838	1.365	1.400	1.413	1.420
E_8	1.031	1.345	1.486	4.635	2.158	2.329	2.393	2.428
E_9	1.022	1.160	1.195	1.145	1.167	1.164	1.162	1.162
E_{new}	***1.147***	***4.557***	***11.708***	***20.541***	***44.652***	***71.136***	***93.530***	***113.557***

Note that: bold and italic values indicate the most efficient estimators (larger values)

Table (5-3): The Relative Efficiencies of the New Ridge Estimator and other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, and ρ =0.99

σ^2 / Estimator	0.1	0.5	1	2	5	10	15	20
E_0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
E_1	2.773	5.887	7.559	12.200	9.597	9.916	10.026	10.082
E_2	4.252	32.436	40.530	43.057	44.119	44.275	44.311	44.326
E_3	1.036	1.177	1.205	1.217	1.217	1.218	1.218	1.218
E_4	6.885	834.861	591.530	127.620	489.425	363.967	334.756	321.731
E_5	3.894	104.877	939.904	88.835	203.692	178.052	170.518	166.919
E_6	2.734	194.033	82.172	30.043	46.463	43.566	42.655	42.209
E_7	1.543	20.850	10.989	5.473	7.311	6.992	6.891	6.841
E_8	2.734	194.033	82.172	30.043	53.368	43.566	42.655	42.209
E_9	1.259	1.243	1.247	1.252	1.250	1.250	1.250	1.250
E_{new}	***29.281***	***969.012***	***2632.097***	***575.669***	***9445.226***	***9038.997***	***8928.474***	***9050.284***

Note that: bold and italic values indicate the most efficient estimators (larger values)

Table (5-4): The Relative Efficiencies of the New Ridge Estimator and Other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, and ρ =0.999

σ^2 / Estimator	0.1	0.5	1	2	5	10	15	20
E_0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
E_1	120.364	578.549	491.802	390.322	433.648	414.212	424.804	423.714
E_2	1451.377	2375.416	2526.525	2693.612	2622.340	2653.127	2635.483	2637.064
E_3	2.649	3.651	3.697	3.704	3.710	3.709	3.710	3.710
E_4	749.557	***4407.829***	***6097.216***	14870.202	8833.783	10671.198	9558.135	9657.352
E_5	87.234	3877.902	5337.146	10860.352	7384.587	8573.880	7869.061	7933.721
E_6	1177.378	1531.287	1900.292	2731.081	2286.248	2460.921	2360.737	2370.283
E_7	3390.981	133.218	163.264	227.097	193.835	207.216	199.620	200.358
E_8	1177.378	1531.287	1900.292	2731.081	2286.248	2460.921	2360.737	2370.283
E_9	1.264	1.266	1.266	1.266	1.266	1.266	1.266	1.266
E_{new}	***1069.011***	1177.187	2288.233	***60671.072***	***9948.137***	***1099227.429***	***26424.533***	***34130.804***

Note that: bold and italic values indicate the most efficient estimators (larger values)

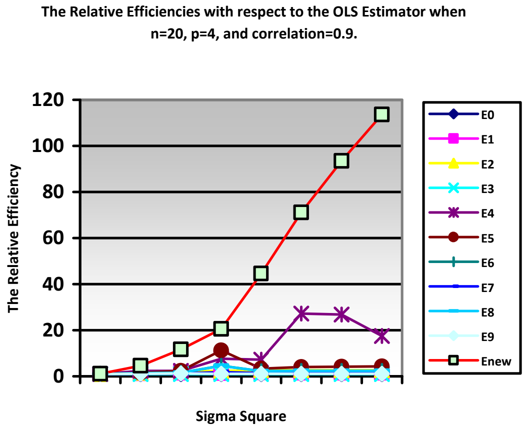

Figure (1-a)

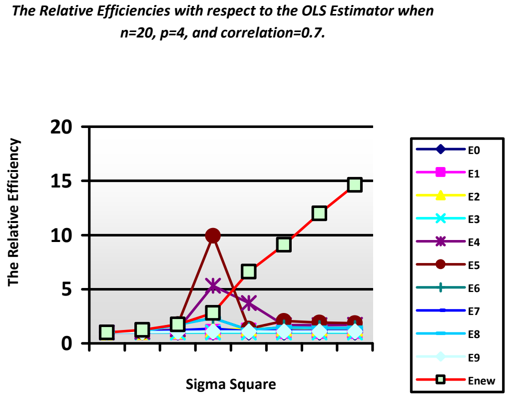

Figure (1-b)

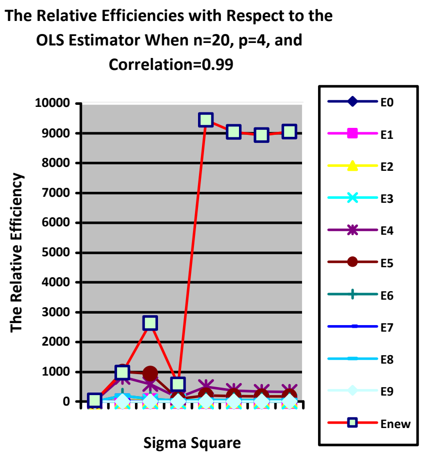

Figure (1-c)

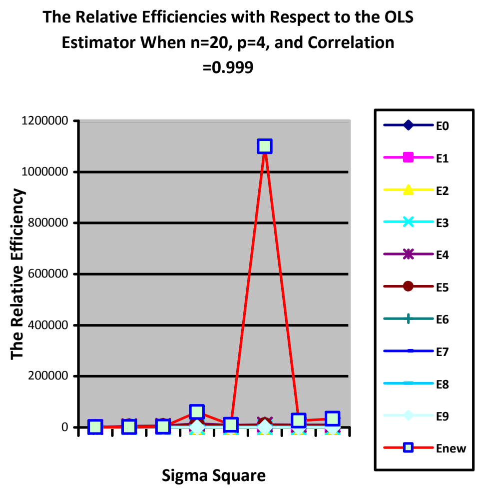

Figure (1-d)

Figure (5-1): The Relative Efficiencies of the New Ridge Estimator and other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, And ρ=0.7

Table (5-5): True and Estimated Regression Parameters Length: OLS Estimator, Ridge Estimators, and the New Ridge Estimator when p=4, ρ=0.9, σ^2=5, and sample sizes n=20, 50, 100, and 200

Estimator Length	N= 20	Sample Size N= 50	N= 100	N= 200
β	1	1	1	1
$\hat{\beta}_0$	**45.75981**	**2.220408**	**3.593403**	**7.431018**
$\hat{\beta}_1$	**32.55879**	**2.198869**	**3.142879**	**7.026633**
$\hat{\beta}_2$	19.0044	2.160005	2.929403	6.538404
$\hat{\beta}_3$	45.0044	2.220076	3.591346	7.429742
$\hat{\beta}_4$	8.786864	2.107313	2.718416	4.991977
$\hat{\beta}_5$	6.961485	2.103691	2.999229	6.267358

$\hat{\beta}_6$	16.11489	1.951489	3.115292	6.725964
$\hat{\beta}_7$	30.23357	2.198114	3.044095	7.045228
$\hat{\beta}_8$	16.92578	2.181219	2.23019	6.355772
$\hat{\beta}_9$	39.92578	2.198482	3.440453	7.272725
$\hat{\beta}_{new}$	***2.355879***	***1.788874***	***1.6677575***	***1.147568***

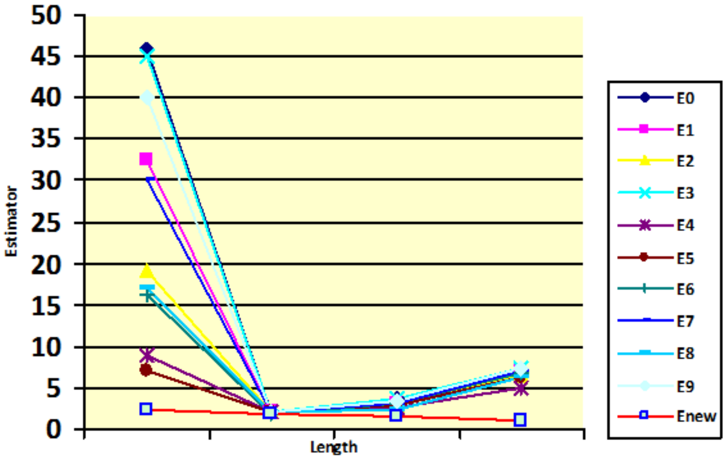

Figure (5 1): Estimated Regression Parameters Length: OLS Estimator, Ridge Estimators, and the New Ridge Estimator When P=4, =0.9,=5, and Sample Sizes N=20, 50 100 and 200

Table (5 6): Bias of New and other Ridge Estimators, When *P=4,* ρ *=0.9, and* σ^2 *=5*

Estimator	n= 20
E_0 Bias	0
E_1 Bias	-0.215877
E_2 Bias	-0.211550
E_3 Bias	-0.2190272
E_4 Bias	-0.1999657
E_5 Bias	-0.1943846
E_6 Bias	-0.209723
E_7 Bias	-0.2152479
E_8 Bias	-0.209723
E_9 Bias	-0.2177059
E_{new} Bias	***-0.1256163***

5.2 When the Length of the True Regression Coefficient=1, And P=10

Four different values of correlation coefficient are used (ρ=0.7, 0.9, 0.99, 0.999), five different sample sizes (n=20, 50, 100, 200, 500), and eight different values of σ^2 (σ^2=0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0). The total experiments = 4×5×8=160 experiments for p=10.

The results of the experiments are shown in Tables A-21:A-28 in Appendix (A). It is found that the average mean squared error (AMSE) of the new proposed estimator (E_{new}) is less than all the ridge estimators and the OLS estimator in 152 experiments for all sample sizes, all levels of σ^2, and all levels of correlation coefficient ρ. In four experiments when n=20, ρ=0.99, it is found that E_8 is better than E_{new} when σ^2=0.5, and E_4 is better than E_{new} when σ^2=0.5, 1.0, 2.0. In other four experiments when n=20, ρ=0.999, and σ^2=0.1, it is found that E_5 is better than E_{new}, and E_4 is better than E_{new} when σ^2=0.5, 1.0, 2.0, 5.0 .

These results indicate that the new ridge estimator, E_{new}, is preferred when σ^2 is greater than one, p=10, and n > 20. Also, as the sample size increases the AMSE of E_{new} decreases and the length of the estimator converges to the true parameter value. It is found that, when ρ=0.99, σ^2=20.0, n=20, 100, 200, 500, …, 5000, the AMSE of E_{new} is 0.096431, 0.0935023, 0.087339, 0.0330145, …, 0.0012215, respectively. This means that the new estimator, E_{new}, has less bias, its AMSE converges to zero as n converges to ∞ and so it is a consistent estimator of β.

5.3 When the Length of the True Regression Coefficient>1, And P=4

Four different values of correlation coefficients are used (ρ=0.7, 0.9, 0.99, 0.999), three different sample sizes (n=20, 50, 200), and eight different values of σ^2 (σ^2=0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0). The total experiments = 4×3×8=96 experiments for p=4, when $\beta'\beta$ >1.

Some of the results of these experiments are shown in Tables 29:31 in Appendix (A-3). It is found that the average mean squared error (AMSE) of the new proposed estimator (E_{new}) is less than all the ridge estimators and the OLS estimator when the length of the true regression coefficient>1, and p=4. Also, as the sample size increases the AMSE of E_{new} decreases and the length of the estimator converges to the true parameter value. It can be concluded, for this case, that the new estimator, $E_{new,}$ is a less biased, has the smallest MSE, and its length converges to the true parameter length, so it is a consistent estimator of β.

5.4 When the Length of the True Regression Coefficient>1, And P=10

Four different values of correlation coefficient are used (ρ=0.7, 0.9, 0.99, 0.99), five different sample sizes (n=20, 100, 500), and eight different values of σ^2 (σ^2=0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0). The total experiments = 4×3×8=96 experiments for p=10, when $\beta'\beta$ >1.

Some of the results of these experiments are shown in Tables 32:34 in Appendix (A-4). It is found that the average mean squared error (AMSE) of the new proposed estimator (E_{new}) is less than all the ridge estimators and the OLS estimator when the length of the true regression coefficient>1, and p=10. Also, as the sample size increases the AMSE of E_{new} decreases and the length of the estimator converges to the true parameter value. It can be concluded, for this case, that the new estimator, $E_{new,}$ is a less biased, has the smallest MSE, and its length converges to the true parameter length, so it is a consistent estimator of β.

(6)CONCLUSIONS

It this paper, a new ridge regression estimator is obtained based on a new proposed ridge parameter. The results of this proposal are shown for four different levels of correlation coefficient ρ=0.7, 0.9, 0.99, 0.99, five different sample sizes n=20, 50, 100, 200, 500, two groups of regressors, p=4 and p=10, and eight different values of σ^2 =0.1, 0.5, 1.0, 2.0, 5.0, 10.0, 15.0, 20.0. Two scenarios are considered: in the first one, it is supposed that the length of the true regression parameter =one, in the second one, it is supposed that the length of the true regression parameter is greater than one. Best results are obtained for the new proposed estimator, in the two scenarios for the two groups, compared to other ridge estimators presented in this paper and to the ordinary least squares estimator (OLS), in the form of:

1. The new estimator has less bias.

2. The new estimator has less average mean squared error (AMSE).

3. The length of the new estimator converges to the length of the true parameter β, as the sample size increases.

4. The AMSE of the new estimator converges to zero as sample size goes to infinity.

It can be concluded that the new proposed estimator is a consistent estimator, while the consistency does not realized for the other ridge estimators and the OLS estimator. The intension in the future is to develop this parameter for fuzzy data.

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Appendix (A)

A.1: Results when the number of regressors, p=4, and the length of regression coeffient vector=1:

Table 1: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=20 n=20

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.001301972	0.03254931	0.1301972	0.520789	3.254931	13.01972	29.29438	52.0789
E_1	0.001299984	0.0319614	0.1253306	0.4885713	2.721425	11.69011	26.15449	46.35791
E_2	0.001299534	0.03131479	0.1171057	0.4244424	2.379052	9.607847	21.4873	38.09751
E_3	0.00130194	0.03253255	0.1300095	0.5192802	3.240012	12.96205	29.16172	51.840760
E_4	0.00129951	0.03088887	0.09913443	0.09794036	0.8713356	7.692589	17.2572	30.57840
E_5	0.00129941	0.03051827	***0.07308563***	***0.05243251***	2.403477	6.295576	15.26509	28.056810
E_6	0.001299145	0.02894654	0.07330893	0.2227293	2.646935	8.87209	20.5251	36.966360
E_7	0.001299525	0.03106229	0.1074113	0.3844263	2.913043	11.13509	25.24547	45.038570
E_8	0.001299145	0.02977892	0.07545782	0.2227293	2.646935	8.87209	20.5251	36.966360
E_9	0.00129948	0.03126955	0.1217069	0.4813283	2.963076	11.94082	26.84777	47.7117
E_{new}	***0.001286582***	***0.02622307***	0.07507272	0.1859515	***0.4912088***	***1.432405***	***2.442077***	***3.563535***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 2: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.9, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=20.N=20

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.004391841	0.109796	0.4391841	1.756736	10.9796	43.91841	98.81642	175.6736
E_1	0.004325399	0.08842872	0.2873421	1.313933	6.748744	28.04782	63.85602	114.1739
E_2	0.004292571	0.07449476	0.2142918	0.7583515	4.213795	16.77908	37.7636	67.16738
E_3	0.004390513	0.1091428	0.4337369	1.730114	10.78214	43.12394	97.02985	172.4999
E_4	0.004285443	0.04523862	0.1903137	0.2311955	1.510883	1.614008	3.689528	10.03154
E_5	0.004285274	0.07296644	0.1848011	0.1569864	3.271598	11.09256	23.44652	40.33694
E_6	0.004216707	0.08909207	0.2955249	0.379024	5.087915	18.86082	41.28683	72.36695
E_7	0.004296359	0.09340385	0.3578259	0.9559575	8.042706	31.3725	69.93293	123.7229
E_8	0.004260388	0.08164239	0.2955249	0.379024	5.087915	18.86082	41.28683	72.36695
E_9	0.004296259	0.0946321	0.3675458	1.534867	9.408363	37.74428	85.00718	151.197
E_{new}	***0.003827522***	***0.02409148***	***0.03751276***	***0.0855249***	***0.2458912***	***0.6173855***	***1.056518***	***1.54701***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 3: The Average Mean Square Error (MSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.99, Different Levels of σ^2 , Number of Regressors P=4, and Sample Size N=20. N=20

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.04740406	1.185102	4.740406	18.96163	118.5102	474.0406	1066.591	1896.163
E_1	0.01709199	0.2013239	0.6271241	1.554293	12.34917	47.80747	106.3842	188.0794
E_2	0.01114965	0.03653699	0.1169613	0.4403858	2.686142	10.70679	24.07078	42.77811
E_3	0.045777	1.00721	3.933721	15.58122	97.35096	389.2166	875.6337	1556.602
E_4	0.00688477	0.00141952	0.0080138	0.1485785	0.2421419	1.302427	3.186177	5.893621
E_5	0.01217352	0.00117935	0.0050435	0.2134471	0.5818112	2.662375	6.255004	11.35978
E_6	0.01733903	0.00610773	0.0576887	0.6311474	2.550642	10.88087	25.00491	44.92279
E_7	0.0307157	0.05683958	0.4313823	3.464708	16.20988	67.79919	154.7899	277.1817
E_8	0.01733903	0.00610773	0.0576888	0.6311474	2.220642	10.88087	25.00491	44.92279
E_9	0.0376627	0.9532697	3.801886	15.14791	94.84308	379.2755	853.298	1516.91
E_{new}	***0.00161891***	***0.0012230***	***0.001801***	***0.0329384***	***0.0125471***	***0.05244394***	***0.1194595***	***0.2095142***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 4: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.999, Different Levels of σ^2 , Number of Regressors P=4, and Sample Size N=20. N=20

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.4808411	12.02103	48.08411	192.3364	1202.103	4808.411	10818.92	19233.64
E_1	0.0039949	0.0207779	0.09777132	0.4927635	2.77207	11.60857	25.46805	45.39297
E_2	0.0003313	0.0050606	0.01903172	0.07140464	0.4584085	1.812356	4.105099	7.29358
E_3	0.181552	3.292766	13.00755	51.92713	324.0201	1296.271	2916.076	5184.171
E_4	0.0006415	***0.0027272***	***0.00788624***	0.01293435	0.1360802	0.4505971	1.131907	1.991606
E_5	0.0055121	0.00309988	0.00900933	0.01770996	0.1627854	0.5608209	1.374868	2.42429
E_6	0.0004084	0.00785028	0.02530354	0.07042502	0.5257973	1.953907	4.582857	8.114492
E_7	***0.0001418***	0.09023563	0.2945176	0.8469363	6.201676	23.20486	54.19765	95.99629
E_8	0.0004084	0.00785028	0.02530354	0.07042502	0.5257973	1.953907	4.582857	8.114492
E_9	0.3803416	9.496249	37.98808	151.9723	949.7664	3799.167	8547.997	15196.46
E_{new}	**0.0004498**	**0.01021166**	**0.02101364**	***0.00317015***	***0.120837***	***0.004374355***	***0.4094271***	***0.5635273***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 5: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=50.N=50

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00002987	0.0007467	0.0029870	0.0119480	0.0746753	0.2987014	0.6720781	1.194805
E_1	0.00002987	0.0007467	0.0029862	0.0119365	0.0730402	0.2941246	0.6536736	1.147559
E_2	0.00002987	0.0007466	0.0029861	0.0119338	0.07407	0.2887883	0.6294727	1.093382
E_3	0.00002987	0.0007468	0.0029870	0.0119480	0.0746722	0.2986477	0.6718378	1.194219
E_4	0.00002987	0.0007467	0.0029861	0.0119335	0.0739701	0.2609593	0.6020502	0.987049
E_5	0.00002987	0.0007467	0.0029861	0.0119339	0.0740427	0.2841718	0.577658	1.076492
E_6	0.00002987	0.0007467	0.0029862	0.0119366	0.0743106	0.2943681	0.6553585	1.153571
E_7	0.00002987	0.0007467	0.0029861	0.0119338	0.0739894	0.2786394	0.6422771	1.154571
E_8	0.00002987	0.0007466	0.0029860	0.0119285	0.0730859	0.2436444	0.5936395	1.105517
E_9	0.00002987	0.0007466	0.0029861	0.0119350	0.0742901	0.2947682	0.6590171	1.166539
E_{new}	***0.00002986***	***0.0007457***	***0.0029722***	***0.0117293***	***0.0674622***	***0.2182264***	***0.4031322***	***0.618877***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 6: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.9, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=50. N=50

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0000677808	0.00169452	0.00677808	0.0271123	0.1694519	0.6778077	1.525067	2.711231
E_1	0.0000677825	0.00169409	0.00676888	0.0269596	0.1646449	0.6264896	1.343298	2.307696
E_2	0.0000677825	0.00169406	0.00676738	0.0269094	0.1610681	0.5794633	1.213331	2.085666
E_3	***0.0000677808***	0.00169452	0.00677802	0.0271112	0.1694071	0.6772271	1.523103	2.707168
E_4	0.0000677825	0.00169406	0.00676723	0.0268975	0.1554758	0.5577377	1.176001	1.934558
E_5	0.0000677825	0.00169407	0.00676783	0.0269237	0.1616794	0.5628869	1.120064	1.769210
E_6	0.0000677825	0.00169409	0.00676889	0.0269604	0.1647685	0.6297948	1.360959	2.34175
E_7	0.0000677825	0.00169407	0.00676780	0.0269185	0.1487681	0.6065172	1.424246	2.558728
E_8	0.0000678256	0.00169399	0.00676381	0.0267269	0.1119333	0.4827649	1.26908	2.355365
E_9	0.0000677825	0.00169409	0.00676874	0.0269652	0.1661358	0.6563606	1.469581	2.606323
E_{new}	0.0000678140	***0.00168633***	***0.00659294***	***0.0240042***	***0.0924808***	***0.1922272***	***0.318503***	***0.480068***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 7: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.99, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=50. N=50

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00067804	0.1695107	0.06780429	0.2712172	1.695107	6.780429	15.25597	27.12172
E_1	0.00067630	0.0159972	0.0567354	0.1733296	0.5716944	2.44679	5.30506	10.03425
E_2	0.00067623	0.0158261	0.05368189	0.1425983	0.4077579	1.211528	2.569216	4.48705
E_3	0.00067803	0.0169447	0.06770363	0.2698165	1.669231	6.642535	14.92888	26.53115
E_4	0.00067623	0.0157890	0.05192779	0.0697023	0.3531829	0.8077313	1.250463	1.695568
E_5	0.00067627	0.0159113	0.05497635	0.1485938	0.3750422	1.305999	2.227549	3.364627
E_6	0.00067630	0.0160002	0.05683558	0.1754316	0.1686889	1.582074	2.895811	4.5818
E_7	0.00067635	0.0161665	0.0546019	0.04830661	0.9178813	4.530437	10.67017	19.34835
E_8	0.00067606	0.0152028	0.03695757	0.01111574	0.3399179	2.252764	5.714696	10.71354
E_9	0.00067634	0.0639699	0.0632664	0.2548042	0.0586113	6.339084	14.25982	25.34834
E_{new}	***0.00064648***	***0.0076979***	***0.00978428***	***0.0105199***	***0.03941252***	***0.1214709***	***0.225952***	***0.3536905***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 8: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.999, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=50. N=50

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00186393	0.0465981	0.1863925	0.7455701	4.659813	18.63925	41.93832	74.5570
E_1	0.00185088	0.0439066	0.1688315	0.6513688	3.943125	15.56617	34.85847	61.8198
E_2	0.00184393	0.0394312	0.1356389	0.4921038	2.951433	11.72081	26.33157	46.78393
E_3	0.00186382	0.0465526	0.1860201	0.7435419	4.645645	18.58152	41.8079	74.32481
E_4	0.00184273	0.0287689	0.0856696	0.2018790	1.166486	4.53107	8.473042	7.436459
E_5	0.00184169	0.0345749	0.0893724	0.3345887	1.521822	6.62815	15.10156	26.94121
E_6	0.00182824	***0.0042919***	0.1054082	0.5386805	3.645210	14.8787	33.68528	60.06525
E_7	0.00184282	0.0384670	0.0990854	0.4671121	3.435554	14.15663	32.11471	57.312
E_8	0.00183067	0.0267370	0.0152753	0.1075381	1.439436	6.85901	16.24063	29.58146
E_9	0.00184237	0.0421149	0.1639121	0.6501758	4.000522	15.91771	35.74972	63.55256
E_{new}	***0.00166014***	***0.0112687***	***0.0123485***	***0.0129048***	***0.026594***	***0.0491396***	***0.0621524***	***0.069879***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 9: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=100. N=100

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0001485	0.0003712	0.0148492	0.0593687	0.3712305	1.484922	3.341074	5.939687
E_1	0.0001485	0.0003712	0.0148437	0.0592942	0.3689148	1.168000	3.292767	5.84204
E_2	0.0001485	0.0003712	0.0148415	0.0592132	0.3649609	1.428114	3.171527	5.595887
E_3	0.0001485	0.0003712	0.0148492	0.0593964	0.3712144	1.484773	3.340625	5.938771
E_4	0.0001485	0.0003712	0.0148413	0.0591940	0.3607920	1.167966	1.731286	4.210402
E_5	0.0001485	0.0003712	0.0148406	0.0591619	0.3568048	1.002404	1.205132	3.734472
E_6	0.0001485	0.0003712	0.0148437	0.0592953	0.3698430	1.468855	3.295765	5.848813
E_7	0.0001485	0.0003712	0.0148413	0.0591993	0.3641866	1.429655	3.187308	5.634355
E_8	0.0001485	0.0003712	0.0483976	0.0591352	0.3612265	1.403856	3.064436	5.330015
E_9	0.0001485	0.0003712	0.0148408	0.0591773	0.3630009	1.420653	3.188824	5.670781
E_{new}	0.0001485	0.0003712	***0.0146257***	***0.0545832***	***0.2483595***	***0.636345***	***1.077658***	***1.578321***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 10: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.9, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=100. N=100

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.000322148	0.00805370	0.03221481	0.1288592	0.8053703	3.221481	7.248332	12.88592
E_1	0.000322160	0.00844136	0.0320720	0.1272548	0.7794911	3.062066	6.825474	11.61357
E_2	0.000322164	0.00804134	0.0319887	0.1254750	0.7282618	2.709755	5.915603	9.952705
E_3	***0.000322148***	0.00805367	0.0322142	0.12885058	0.8051570	3.219971	7.244292	12.87659
E_4	0.000322161	0.00804116	0.0319755	0.1247044	0.6055032	0.532165	3.611729	5.671054
E_5	0.000322161	0.00804044	0.0319492	0.1238428	0.5303892	0.7250734	3.070994	8.692492
E_6	0.000322160	0.00804415	0.0320727	0.1272760	0.7803164	3.069653	6.848889	11.72702
E_7	0.000322161	0.00804149	0.0319972	0.1259268	0.7547663	2.881215	6.28784	9.997871
E_8	0.000322162	0.00803961	0.0319457	0.1250671	0.7078277	2.412017	4.897821	6.235111
E_9	0.000322162	0.00804067	0.0319686	0.1253523	0.7596828	3.025894	6.811824	12.18178
E_{new}	0.000322565	***0.00769946***	***0.0267237***	***0.0724526***	***0.1803833***	***0.3564926***	***0.5708177***	***0.817179***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 11: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.99, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=100. N=100

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00213397	0.05334924	0.2133970	0.8535878	5.334924	21.3397	48.01432	85.35878
E_1	0.00209661	0.04101017	0.1242716	0.3612158	2.102281	9.211857	2.30795	38.3903
E_2	0.00208577	0.03441352	0.0849726	0.2385236	1.292693	5.087571	11.4335	20.3306
E_3	0.00213384	0.05327793	0.2126413	0.8485825	5.297528	21.18717	47.67067	84.74803
E_4	0.00280849	0.02663046	0.0613640	0.1622636	0.159550	2.032351	5.080605	9.3397828
E_5	0.00282669	0.03361211	0.0573989	0.2454715	1.209649	3.899603	8.075005	13.73769
E_6	0.00209667	0.00725600	0.0291457	0.2676154	2.259481	9.768327	22.50954	40.48283
E_7	0.00207394	0.04386311	0.1179740	0.2567887	1.245221	5.221951	12.11085	21.90717
E_8	0.00208682	0.03047317	0.0496972	0.0722866	0.308034	1.317529	3.098559	5.651137
E_9	0.00293307	0.05036754	0.2001561	0.7985253	4.984800	19.93265	44.84392	79.71861
E_{new}	***0.00142107***	***0.00354065***	***0.0047406***	***0.0109548***	***0.044709***	***0.1365927***	***0.266258***	***0.429162***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 12: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.999, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=100. N=100

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.01832716	0.4581790	1.8327160	7.3308640	45.81790	183.2716	412.3611	733.0864
E_1	0.00866576	0.0631142	0.1730840	0.5463125	2.902396	10.95437	24.16328	42.5292
E_2	0.00671231	0.0144207	0.0362101	0.1241013	0.737353	2.923491	6.564602	11.66068
E_3	0.01824756	0.4386893	1.0727438	6.8674150	42.82932	171.2510	385.2802	684.9168
E_4	0.00614948	0.0005563	0.0092083	0.0459643	0.280698	1.083052	2.398860	4.228066
E_5	0.00661018	0.0003344	0.0069226	0.0226886	0.253684	1.205225	2.865600	5.234870
E_6	0.00327267	0.0660429	0.1859054	0.5991341	3.235815	12.29028	27.17017	47.87553
E_7	0.01165295	0.0577704	0.1606792	0.2591787	1.187236	4.302275	9.376531	16.40983
E_8	0.00579057	0.0097729	0.0136847	0.0303603	0.137366	0.500146	1.092842	1.915383
E_9	0.01729677	0.4279316	1.7117750	6.8473230	42.79685	171.189	385.1765	684.7593
E_{new}	***0.00043839***	***0.0000566***	***0.0000746***	***0.0003881***	***0.005782***	***0.031961***	***0.078795***	***0.144505***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 13: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=200. N=200

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.000201614	0.00504034	0.0201614	0.08064558	0.5040349	2.016140	4.536314	8.064558
E_1	0.000201616	0.00503742	0.0201221	0.08020357	0.4963676	1.965811	4.429124	7.888418
E_2	0.000201616	0.00503584	0.0200883	0.07963083	0.4865996	1.928200	4.331254	7.697010
E_3	0.000201614	0.00504343	0.0201613	0.08064429	0.5040122	2.016024	4.536045	8.064076
E_4	0.000201616	0.00503554	0.0200699	0.07663881	0.4674939	1.822957	4.176425	7.145317
E_5	0.000201617	0.00535484	0.0200769	0.07901271	0.4823069	1.917507	4.254924	7.484216
E_6	0.000201617	0.00532567	0.0199160	0.07096876	0.4851137	1.969084	4.444400	7.911862
E_7	0.000201616	0.00503551	0.0200778	0.07976679	0.4955692	1.973521	4.430293	7.865412
E_8	0.000201617	0.00503309	0.0200030	0.07911471	0.4898925	1.922432	4.283169	7.570701
E_9	0.000201616	0.00503565	0.0200889	0.07978202	0.4954518	1.984216	4.467112	7.944091
E_{new}	0.000201703	***0.00489368***	***0.0180482***	***0.05722531***	***0.2024546***	***0.516094***	***0.958157***	***1.499590***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 14: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.9, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=200. N=200

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00067186	0.01679643	0.06718574	0.2687429	1.679643	6.718574	15.11679	26.87429
E_1	0.00067147	0.01669695	0.06611751	0.2599006	1.578658	6.201809	13.84614	24.67962
E_2	0.00067134	0.01661835	0.06479217	0.2461320	1.457989	5.764284	12.94515	23.00191
E_3	0.00067185	0.01679621	0.06718263	0.2687122	1.679326	6.717194	15.11365	26.86869
E_4	0.00067134	0.01658877	0.06246692	0.2257995	1.067246	5.083716	11.59458	20.69284
E_5	0.00067130	0.01659887	0.06372528	0.1956916	1.393609	5.229984	11.28981	19.48525
E_6	0.00067123	0.01633349	0.04936038	0.2035631	1.505228	6.174489	13.98465	24.9371
E_7	0.00067134	0.01664856	0.06588793	0.2600515	1.584699	6.225797	13.89925	24.60444
E_8	0.00067124	0.01656912	0.06526809	0.2492828	1.415334	5.303797	11.61036	20.3365
E_9	0.00067131	0.01664289	0.06572877	0.2622228	1.640423	6.566343	14.77830	26.27627
E_{new}	***0.00065435***	***0.01233065***	***0.02960917***	***0.0561788***	***0.138257***	***0.306018***	***0.497269***	***0.7242217***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 15: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.99, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=200. N=200

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00709020	0.1772551	0.7090205	2.836082	17.72551	70.90205	159.5296	283.6082
E_1	0.00685263	0.1385749	0.4778348	1.684325	9.476911	36.35991	80.61086	142.2297
E_2	0.00673132	0.0997582	0.2919431	1.013809	6.046716	24.03385	54.02221	96.01193
E_3	0.00708973	0.1770912	0.7078548	2.830317	17.68703	70.74677	159.1797	282.986
E_4	0.00670801	0.0473140	0.1314608	0.428614	3.584388	14.71344	33.2253	59.12147
E_5	0.00670653	0.0569352	0.1897570	0.698077	2.983051	9.624539	19.87897	33.75743
E_6	0.00648279	0.0180476	0.2115146	1.198171	8.699249	36.30375	82.78478	148.1423
E_7	0.00691629	0.1503723	0.5111237	1.716309	9.169510	34.49746	75.98838	133.647
E_8	0.00675916	0.1061021	0.2749201	0.744195	3.369945	12.10294	26.17094	45.60544
E_9	0.00693436	0.1723012	0.6892049	2.757148	17.23405	68.9391	155.1153	275.7625
E_{new}	***0.00228071***	***0.0026969***	***0.0035010***	***0.006718***	***0.019655***	***0.0473993***	***0.080923***	***0.1191897***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 16: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.999, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=200. N=200

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.07133315	1.7833290	7.133315	28.53326	178.3329	713.3315	1604.996	2853.326
E_1	0.02497869	0.1929503	0.5842366	1.989275	11.19431	43.17255	95.94686	170.7071
E_2	0.01209794	0.0540308	0.1859846	0.713989	4.408615	17.6013	39.58768	70.36776
E_3	0.07098528	1.7496960	6.98519	27.92517	174.5031	697.9942	1570.479	2791.956
E_4	0.00585912	0.0011906	0.0501487	0.245593	1.62276	6.555183	14.78411	26.30955
E_5	0.00769152	0.0293535	0.0749359	0.1961704	0.877555	3.091520	6.655056	11.55809
E_6	0.00059911	0.2115298	0.6608932	2.297959	13.12456	50.88982	113.3066	200.3749
E_7	0.03775971	0.0429755	0.2164924	1.081941	7.848144	33.02925	75.59068	135.5324
E_8	0.01431041	0.0038797	0.0205189	0.1079463	0.813104	3.468690	7.975196	14.33268
E_9	0.06930183	1.732745	6.930839	27.72311	173.2686	693.0731	1559.414	2772.29
E_{new}	***0.00008125***	***0.0004656***	***0.0009218***	***0.0014413***	***0.001735***	***0.001163***	***0.000827***	***0.001265***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 17: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=500. N=500

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0000321	0.0008025	0.0032100	0.01284007	0.08025045	0.3210018	0.7222540	1.284007
E_1	0.0000321	0.0008025	0.0032099	0.01283758	0.08018483	0.3204392	0.7205021	1.280269
E_2	0.0000321	0.0008025	0.0032098	0.01283625	0.08009556	0.3194449	0.7177216	1.275210
E_3	0.0000321	0.0008025	0.0032100	0.01284007	0.08025037	0.3210010	0.7222518	1.284003
E_4	0.0000321	0.0008025	0.0032098	0.01283568	0.07888971	0.3186076	0.7132577	1.25834
E_5	0.0000321	0.0008025	0.0032098	0.01283639	0.08005577	0.3176676	0.7087371	1.232005
E_6	0.0000321	0.0008025	0.0032097	0.01282944	0.07886225	0.3189058	0.7190861	1.279303
E_7	0.0000321	0.0008025	0.0032098	0.01283568	0.08011708	0.3203341	0.7206377	1.281018
E_8	0.0000321	0.0008025	0.0032097	0.01283131	0.07996417	0.3201116	0.7199502	1.279721
E_9	0.0000321	0.0008025	0.0032098	0.01283690	0.08015839	0.3203060	0.7206532	1.281053
E_{new}	0.0000321	***0.0008023***	***0.0031982***	***0.01259413***	***0.07173560***	***0.2398988***	***0.4721807***	***0.760993***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 18: The Average Mean Square Error (Amse) of the Proposed and other Ridge Regression Estimators Compared to Ols Estimators with Level of Multicollinearity ρ =0.9, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=500. N=500

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00008426	0.00210656	0.00842626	0.03370506	0.2106566	0.8426265	1.895910	3.370506
E_1	0.00008426	0.00210627	0.00842222	0.03365724	0.2097475	0.8362328	1.877819	3.334063
E_2	0.00008426	0.00210622	0.00842048	0.03361596	0.2083995	0.8277695	1.858657	3.301962
E_3	0.00008426	0.00210657	0.00842626	0.03370502	0.2106555	0.8426190	1.895891	3.370471
E_4	0.00008426	0.00210621	0.00842000	0.03357907	0.2070497	0.8171687	1.799154	3.059064
E_5	0.00008426	0.00210623	0.00842000	0.03361168	0.2065475	0.8216968	1.830869	3.220214
E_6	0.00008426	0.00210607	0.00841424	0.03320661	0.2035557	0.8309422	1.875367	3.337941
E_7	0.00008426	0.00210622	0.00842082	0.03364148	0.2099814	0.8393550	1.887869	3.355455
E_8	0.00008426	0.00210609	0.00841709	0.03358527	0.2097827	0.8368265	1.879880	3.338694
E_9	0.00008426	0.00210624	0.00842161	0.03365123	0.2098627	0.8395011	1.889037	3.358460
E_{new}	***0.00008415***	***0.00208207***	***0.00804968***	***0.02872800***	***0.1176828***	***0.2949070***	***0.506439***	***0.7468482***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 19: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.99, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=500. N=500

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00083673	0.0215918	0.08636731	0.3454692	2.159183	8.636731	19.43264	34.54692
E_1	0.00086280	0.0213311	0.08371427	0.3242054	1.926430	7.685723	17.43833	31.12132
E_2	0.00086267	0.0211716	0.08112266	0.3014185	1.766109	6.970750	15.64770	27.79837
E_3	0.00086367	0.0215916	0.08636449	0.3454437	2.158939	8.635686	19.43027	34.54268
E_4	0.00086266	0.0210845	0.07059556	0.2839207	1.390590	4.142698	11.25877	20.89968
E_5	0.00086270	0.0211724	0.08000517	0.2671355	1.757502	6.622738	14.37386	25.02410
E_6	0.00086240	0.0202147	0.04421653	0.2628452	1.898402	7.782455	17.62863	31.43810
E_7	0.00086282	0.0214705	0.08546208	0.3392426	2.096453	8.331414	18.69564	33.18850
E_8	0.00086261	0.0213343	0.08379524	0.3254682	1.949654	7.614644	16.96770	30.00730
E_9	0.00086277	0.0214925	0.08592660	0.3436875	2.148131	8.592737	19.33384	34.37145
E_{new}	***0.00079822***	***0.0087976***	***0.01150526***	***0.0126708***	***0.023203***	***0.052075***	***0.088774***	***0.130416***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 20: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.999, Different Levels of σ^2, Number of Regressors P=4, and Sample Size N=500. N=500

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0088995	0.2224883	0.8899534	3.559814	22.24883	88.99534	200.2395	355.9814
E_1	0.0083657	0.1415017	0.4336342	1.377871	9.616493	40.24028	91.83835	171.1125
E_2	0.0081723	0.0967216	0.2659443	0.907934	5.391409	21.41480	48.12728	85.62371
E_3	0.0088991	0.2223503	0.8890401	3.555458	22.22011	88.87962	199.9788	355.5183
E_4	0.0081213	0.0646722	0.1664063	0.184006	1.573236	7.929437	18.80181	37.95594
E_5	0.0081935	0.0614030	0.2514438	0.945654	4.297730	14.88832	31.79380	46.16003
E_6	0.0075472	0.0235389	0.2690909	1.447296	10.31020	42.82390	97.51457	180.3957
E_7	0.0087339	0.1955035	0.7220826	2.695786	15.90300	62.16807	138.7451	239.2406
E_8	0.0083687	0.1443289	0.4562547	1.507027	8.078532	30.44793	67.09268	110.3848
E_9	0.0088522	0.2212573	0.8850309	3.540136	22.12591	88.50375	199.1335	354.0156
E_{new}	***0.0007658***	***0.0003637***	***0.0009117***	***0.002533***	***0.008653***	***0.023103***	***0.041899***	***0.011337***

Note that: bold and italic values indicate the smallest values of the AMSE.

A. 2: Results When the Number of Regressors, P=10, and the Length of Regression Coeffient Vector=1:

Table 21: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Level of Multicollinearity ρ =0.7, Different Levels of σ^2, Number of Regressors P=10, and Sample Size N=20. N=20

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00186392	0.04659813	0.1863925	0.7455701	4.659813	18.63925	41.93832	74.55701
E_1	0.00185088	0.04390664	0.1688315	0.6513688	3.943125	15.56617	34.85847	61.81980
E_2	0.00184392	0.39431118	0.1356389	0.4921038	2.951433	11.72081	26.33157	46.78393
E_3	0.00186382	0.04655256	0.1860201	0.7435419	4.645645	18.58152	41.80790	74.32481
E_4	0.00184273	0.02876893	0.0856696	0.2018790	1.166486	4.531073	8.473042	7.436459
E_5	0.00184169	0.03457494	0.0893723	0.3345887	1.521822	6.628150	15.10156	26.94121
E_6	0.00182824	0.04291950	0.1054082	0.5386805	3.645210	14.87870	33.68528	60.06525
E_7	0.00184282	0.03846700	0.0998537	0.4671121	3.435554	14.15663	32.11471	57.31200
E_8	0.00183066	0.02673701	0.0152753	0.1075381	1.439436	6.859010	16.24063	29.58146
E_9	0.00184236	0.04211494	0.1639121	0.6501758	4.000522	15.91771	35.74972	63.55256
E_{new}	***0.00166014***	***0.0112687***	***0.0123485***	***0.0129048***	***0.026594***	***0.049139***	***0.062152***	***0.069879***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 22: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Different Levels of Multicollinearity, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=10, and ρ =0.9.n=20

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00640132	0.1600329	0.6401317	2.560527	16.00329	64.01317	144.0296	256.0527
E_1	0.00604837	0.123102	0.4485611	1.680268	10.00869	37.05061	87.94821	155.8663
E_2	0.00574065	0.06828334	0.2139315	0.7843151	4.767259	19.09786	42.68014	75.85037
E_3	0.00639751	0.1590626	0.6346772	2.536214	15.84616	63.36789	142.6066	253.5218
E_4	0.00564231	0.02559599	0.2528392	0.0423532	0.7550744	2.064498	5.583733	12.07334
E_5	0.00561785	0.01714462	0.3485732	0.212976	1.815892	8.064498	17.19179	30.65117
E_6	0.00478198	0.01830833	0.1876571	0.9833899	6.959173	28.65508	65.72051	117.5094
E_7	0.00586271	0.03721923	0.1909744	1.214263	8.873899	36.45846	84.0064	150.1956
E_8	0.00530821	0.00235687	0.01363633	0.1664887	1.700646	7.745491	18.35996	33.34516
E_9	0.00597829	0.1366087	0.5383501	2.132776	13.25465	52.81420	118.9975	211.4862
E_{new}	***0.00246964***	***0.00229737***	***0.00359997***	***0.0120793***	***0.0418981***	***0.226712***	***0.0776409***	***0.0831682***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 23: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Different Levels of Multicollinearity, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=10, and ρ =0.99

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0702117	1.755292	7.02117	28.08468	175.5292	702.117	1579.763	2808.468
E_1	0.0175371	0.2031917	0.7087304	2.634389	15.72919	6.94702	138.6563	245.8572
E_2	0.0039718	0.0298866	0.1119789	0.4403659	2.738867	10.94745	24.62822	43.78118
E_3	0.0667150	1.569773	6.25162	24.9774	156.056	624.192	1404.418	2496.733
E_4	0.0001819	0.0023198	***0.0076697***	***0.0335239***	0.1621736	0.961899	2.356468	4.352661
E_5	0.0007749	0.0024846	0.01919866	0.0914746	0.6066731	2.456902	5.546341	9.874972
E_6	0.0007906	0.0276668	0.1554339	0.7243382	4.937735	20.31553	46.13764	82.40406
E_7	0.0065391	0.07385451	0.5082664	2.539337	17.89911	74.37609	169.4484	303.116
E_8	0.0002259	***0.0016656***	0.0143643	0.0809677	0.6126854	2.606006	5.983321	10.74464
E_9	0.0547248	1.352526	5.40395	21.6041	134.8929	539.8757	1214.678	2159.391
E_{new}	***0.0001349***	0.007800	0.02599483	0.05362537	***0.0818685***	***0.091985***	***0.095033***	***0.096431***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 24: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to OLS Estimators with Different Levels of Multicollinearity, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=10, and ρ =0.999

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.7154428	7.8807	71.54428	286.1771	1788.607	7154.428	16097.46	28617.71
E_1	0.00346792	0.0524369	0.1955745	0.7549419	4.617759	18.33816	41.16153	73.0682
E_2	0.00022271	0.0044170	0.0175319	0.0700061	0.4373827	1.749524	3.936473	6.992606
E_3	0.3276812	7.587715	30.26983	120.9973	756.0862	3024.256	6804.536	12096.92
E_4	0.00007502	***0.0002882***	***0.0006445***	***0.0037271***	***0.031915***	0.138037	0.3181962	0.5709432
E_5	***0.00006971***	0.0006793	0.0029582	0.0122167	0.077550	0.311620	0.702166	1.246791
E_6	0.00010344	0.0078303	0.0352792	0.149556	0.9674472	3.914146	8.840278	15.73826
E_7	0.00021416	0.0450950	0.2212139	0.9756055	6.456684	26.31837	59.58828	106.2347
E_8	0.00019137	0.0005549	0.00275320	0.0123062	0.082188	0.336066	0.761703	1.356503
E_9	0.5397417	13.48968	53.95668	215.8227	1348.877	5395.49	12139.84	21581.88
E_{new}	0.0028497	0.0449651	0.0710332	0.087096	0.095993	***0.0983462***	***0.098996***	***0.099286***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 25: The Average mean square error (AMSE) of the proposed and other ridge regression estimators compared to the OLS estimator with, different levels of σ^2, sample size n=50, number of regressors p=10, and ρ =0.9

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0008929	0.0223223	0.0892892	0.3571572	2.232232	8.928929	20.09009	35.71572
E_1	0.0008921	0.0211130	0.0798183	0.3045648	1.979124	7.995198	18.04383	32.12511
E_2	0.0008917	0.0201053	0.0701949	0.2480181	1.45291	5.744297	12.89471	22.9045
E_3	0.0008928	0.0223170	0.0892348	0.3567961	2.229459	8.91745	20.06409	35.66938
E_4	0.0008916	0.0190383	0.0590874	0.1974346	0.9787719	3.602299	7.732132	13.3659
E_5	0.0008917	0.0200969	0.0667342	0.1862728	0.8485503	2.682315	5.310875	8.780985
E_6	0.0008908	0.0049448	0.0681851	0.3057299	1.983429	8.009838	18.07507	32.17922
E_7	0.0008916	0.0210813	0.0809434	0.3069778	1.774457	6.780018	14.97345	26.35436
E_8	0.0008910	0.0200980	0.0677490	0.204098	0.8503099	2.741969	5.677051	9.660939
E_9	0.0008917	0.0210619	0.0827913	0.326949	2.016185	8.027278	18.03322	32.03404
E_{new}	***0.0008642***	***0.0053813***	***0.0054918***	***0.0049046***	***0.0112504***	***0.0290290***	***0.0439885***	***0.05474385***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 26: The Relative Efficiencies of the New Ridge Estimator and other Ridge Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=100, Number of Regressors P=10, and ρ =0.99

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.009918	0.2479649	0.9918594	3.967438	24.79649	99.18594	223.1684	396.7438
E_1	0.008523	0.1607362	0.5933719	2.262459	13.69801	54.18399	121.4533	215.5058
E_2	0.007021	0.07961139	0.2739694	1.045847	6.449141	25.75109	57.92385	102.9674
E_3	0.009913	0.2471281	0.9876086	3.949225	24.68043	98.72055	222.1208	394.8813
E_4	0.006620	0.04316955	0.1005261	0.2640045	1.095237	2.76251	4.283434	4.425767
E_5	0.006764	0.03349259	0.1386097	0.8069086	5.077689	19.07419	41.9824	73.80256
E_6	0.004278	0.06096377	0.3453871	1.558388	10.35687	42.22196	95.58853	170.4566
E_7	0.008811	0.1408368	0.3927574	1.195481	6.416088	24.78183	55.26226	97.86034
E_8	0.006879	0.03868655	0.0540803	0.1038533	0.4476081	1.649876	3.636069	6.405937
E_9	0.009094	0.2264017	0.9052688	3.6208500	22.63038	90.52222	203.6757	362.0907
E_{new}	***0.000635***	***0.00077046***	***0.0038591***	***0.0157056***	***0.0513463***	***0.07821545***	***0.0885466***	***0.0935023***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 27: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=200, Number of Regressors P=10, and ρ=0.99.

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.00403178	0.1007947	0.4031789	1.612716	10.07947	40.31789	90.71525	161.2716
E_1	0.00383721	0.08157901	0.3044439	1.161618	7.018894	27.7314	62.13161	110.2195
E_2	0.00361954	0.04719708	0.1499324	0.5503305	3.349363	13.34986	30.02023	53.36051
E_3	0.00403155	0.1007276	0.4027516	1.610715	10.06637	40.26514	90.59645	161.0603
E_4	0.00358496	0.02674539	0.02095405	0.1338564	1.225633	5.580837	12.94558	23.32657
E_5	0.00354883	0.02169231	.03306397	0.2420888	0.834509	3.119753	7.687958	14.2704
E_6	0.00319262	0.02055412	0.1564542	0.7531989	5.14265	21.12206	47.93102	85.56952
E_7	0.00385607	0.08532996	0.2956896	1.005368	5.38553	20.22748	44.51077	78.23784
E_8	0.00364175	0.0457762	0.1147147	0.301658	1.29237	4.430462	9.438022	16.31649
E_9	0.00386673	0.09584503	0.3831442	1.53237	9.577075	38.30834	86.1939	153.2337
E_{new}	***0.00052908***	***0.00035318***	***0.00132261***	***0.00641367***	***0.03154882***	***0.06280892***	***0.07873009***	***0.08733955***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 28: The Average mean square error (AMSE) of the proposed and other ridge regression estimators compared to the OLS estimator with, different levels of σ^2, sample size n=500, number of regressors p=10, and ρ=0.999

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.01243042	0.3107606	1.243042	4.97217	31.07606	124.3042	279.6846	497.217
E_1	0.00946522	0.2118481	0.568611	2.12358	12.66746	49.83544	111.4968	197.6515
E_2	0.00761055	0.1771992	0.221167	0.833353	5.112081	20.38596	45.83832	81.46915
E_3	0.01242827	0.3105456	1.241243	4.964526	31.0274	124.109	279.245	496.4354
E_4	0.00700028	0.1814366	0.043764	0.07286912	0.4717167	2.948709	7.665147	14.4316
E_5	0.00713579	0.1759865	0.109507	0.3104066	2.364861	9.850688	22.40629	40.03102
E_6	0.00387449	0.2131191	0.272640	1.31036	8.982466	36.96139	83.93008	149.8885
E_7	0.01162505	0.2851753	0.835872	3.113284	18.47152	72.47815	161.9953	287.0228
E_8	0.00849413	0.2109714	0.258941	0.8106379	4.238316	15.87531	34.91967	61.37165
E_9	0.01223909	0.3089240	1.223516	4.894036	30.58763	122.3504	275.2884	489.4015
E_{new}	***1.350813e-05***	***0.2140470***	***0.00210712***	***0.01017978***	***0.03829805***	***0.06273744***	***0.0734448***	***0.07916712***

Note that: bold and italic values indicate the smallest values of the AMSE.

A. 3: Results When the Number of Regressors, P=4, and the Length of Regression Coeffient Vector > 1:

The results are shown for three selected sample sizes n=20, 50, 200, when the correlation coefficient ρ **=0.9**, 0.99, 0.999, respectively.

Table 29: The Average Mean Square Error (AMSE) of the Proposed and Other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=4, and ρ=0.9.

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.004391841	0.109796	0.4391841	1.756736	10.9796	43.91841	98.81642	175.6736
E_1	0.004369547	0.1007072	0.3531382	1.146087	6.161072	27.00565	62.35459	112.2119
E_2	0.004366707	0.09600601	0.2984952	0.8591235	4.356625	16.86816	37.79573	67.14234
E_3	0.004391509	0.109592	0.4366029	1.735175	10.79173	43.13084	97.03356	172.5004
E_4	.004366482	0.09241769	0.1833925	0.7578778	2.754417	5.882511	7.399627	5.684788
E_5	0.00436762	0.09686104	0.289747	0.7480071	4.263437	13.18732	26.61916	44.57909
E_6	0.004369566	0.1008304	0.355836	1.179204	5.765643	20.32032	43.52182	75.3752
E_7	0.004367446	0.09945755	0.3736527	1.430911	8.364388	32.16033	71.19666	125.465
E_8	0.004360873	0.08859527	0.3265108	1.179204	5.765643	20.32032	43.52182	75.3752
E_9	0.004369619	0.1023728	0.3785827	1.471044	9.354405	37.63734	84.84749	150.9846
E_{new}	***0.004167532***	***0.04551503***	***0.06728569***	***0.1040409***	***0.2856822***	***0.7088045***	***1.206705***	***1.759617***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 30: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator With, Different Levels of σ^2, Sample Size N=50, Number of Regressors P=4, and ρ=0.99.

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0006780429	0.002712172	0.01695107	0.06780429	0.2712172	1.695107	6.780429	15.25597
E_1	0.0006763046	0.002684019	0.01599726	0.0567354	0.1733296	0.5716944	2.044679	5.30506
E_2	0.0006762336	0.002681763	0.01582613	0.05368189	0.1425983	0.4077579	1.211528	2.569216
E_3	0.0006780337	0.002712014	0.01694468	0.06770363	0.2698165	1.669231	6.642535	14.92888
E_4	0.0006762312	0.002681598	0.01578902	0.05192779	0.06797023	0.3531829	0.8077313	1.250463
E_5	0.0006762716	0.002682953	0.01591127	0.05497635	0.1485938	0.3750422	1.305999	2.227549
E_6	0.0006763049	0.002684036	0.01600021	0.05683558	0.1754316	0.6186889	1.582074	2.895811
E_7	0.0006763574	0.00268701	0.01616646	0.0546019	0.04830661	0.9178813	4.530437	10.67017
E_8	0.0006760671	0.00267581	0.0152028	0.03695757	0.01111574	0.3399179	2.252764	5.714696
E_9	0.0006763412	0.00268696	0.01639699	0.06432664	0.2548042	1.586113	6.339084	14.25982
E_{new}	***0.0006464852***	***0.002270277***	***0.007697925***	***0.009784279***	***0.01051599***	***0.0394125***	***0.1214709***	***0.225952***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 31: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=200, Number of Regressors P=4, and ρ=0.999.

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.07133315	1.783329	7.133315	7.133315	178.3329	713.3315	1604.996	2853.326
E_1	0.04171003	0.295963	0.7717271	0.7717271	12.0141	44.77672	98.33496	172.6892
E_2	0.03161101	0.08564077	0.2160972	0.2160972	4.43955	17.63425	39.62267	70.40478
E_3	0.07122924	1.758676	6.998786	6.998786	174.52	698.0125	1570.498	2791.977
E_4	0.02890682	0.02711149	0.004740867	0.00474086	1.570744	6.490791	14.70727	26.22024
E_5	0.02982484	0.01883844	0.1174013	0.1174013	1.104753	3.510109	7.263307	12.36587
E_6	0.04188101	0.3126362	0.846039	0.846039	13.94427	52.49764	115.7022	203.5581
E_7	0.05065493	0.08099297	0.1718524	0.1718524	7.100189	31.39153	73.06301	132.1148
E_8	0.02666477	0.00843162	0.01550563	0.01550563	0.7169162	3.252176	7.637797	13.87426
E_9	0.06937133	1.732826	6.930967	6.930967	173.2691	693.0741	1559.415	2772.292
E_{new}	***0.000253598***	***0.00131782***	***0.00388335***	***0.00388335***	***0.0151792***	***0.01960894***	***0.02003059***	***0.01864406***

Note that: bold and italic values indicate the smallest values of the AMSE.

A. 4: Results When the Number of Regressors, P=10, and the Length of Regression Coeffient Vector > 1:

The results are shown for three selected sample sizes n=20, 100, 500, when the correlation coefficient ρ **=0.9**, 0.9, 0.99, respectively.

Table 32: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=20, Number of Regressors P=10, and ρ=0.9

σ^2 / Estimator	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.006401317	0.1600329	0.6401317	2.560527	16.00329	64.01317	144.0296	256.0527
E_1	0.006334939	0.1413725	0.494938	1.684556	8.816334	36.09858	83.16519	149.53
E_2	0.006320461	0.1253753	0.3500068	0.9718194	4.964464	19.16569	42.83994	75.98964
E_3	0.006400886	0.1598103	0.6376322	2.542213	15.85405	63.38928	142.6142	253.529
E_4	0.006319414	0.1131848	0.2445138	0.3859103	2.74986	8.216151	15.87727	25.67993
E_5	0.006322346	0.1253241	0.3125189	0.7325246	2.548781	6.721875	16.322	29.78489
E_6	0.006334958	0.1414371	0.495918	1.693449	8.911424	32.9012	71.83006	125.6976
E_7	0.006324088	0.1351863	0.5009058	1.902063	10.91694	41.36225	90.92604	159.5823
E_8	0.006296933	0.0971850	0.3491549	1.089628	4.056125	12.43965	25.23618	42.46311
E_9	.006326711	0.1408261	0.5291802	2.058338	13.05565	52.51575	118.3877	210.6713
E_{new}	***0.003456796***	***0.0057114***	***0.02427827***	***0.1279236***	***0.4759704***	***0.7369685***	***0.8403578***	***0.8919165***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 33: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=100, Number of Regressors P=10, and ρ=0.9.

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.0009675651	0.02418913	0.09675651	0.3870261	2.418913	1] 9.675651	21.77022	38.70261
E_1	0.000966847	0.02400719	0.09495642	0.3721859	2.246359	8.781982	19.56213	34.58262
E_2	.0009667741	0.0239132	0.09316961	0.3482402	1.920817	7.300643	16.2182	28.69005
E_3	0.0009675643	0.02418885	0.09675271	0.3869809	2.41822	9.672088	21.76172	38.68718
E_4	0.0009667726	0.02389966	0.09240883	0.3210245	1.507403	5.676684	11.57347	18.818
E_5	0.0009667707	0.02390205	0.09272512	0.3336512	1.507503	4.756238	11.79404	22.32608
E_6	0.0009668471	0.02400749	0.09496439	0.3723275	2.249493	8.803104	19.61952	34.69506
E_7	0.0009667777	0.02394345	0.09408104	0.3645593	2.121564	7.71241	16.52775	28.71993
E_8	0.0009667057	0.02384915	0.09215817	0.3341834	1.42871	3.888927	7.387329	12.16101
E_9	0.000966774	0.02395296	0.09444891	0.3715712	2.286041	9.089537	20.43092	36.32123
E_{new}	***0.0008053082***	***0.008711011***	***0.01348532***	***0.01509018***	***0.07900867***	***0.2683144***	***0.4257762***	***0.5370761***

Note that: bold and italic values indicate the smallest values of the AMSE.

Table 34: The Average Mean Square Error (AMSE) of the Proposed and other Ridge Regression Estimators Compared to the OLS Estimator with, Different Levels of σ^2, Sample Size N=500, Number of Regressors P=10, and ρ=0.99

Estimator \ σ^2	0.1	0.5	1.0	2.0	5.0	10.0	15.0	20.0
E_0	0.001346582	0.03366455	0.1346582	0.5386328	3.366455	13.46582	30.2981	53.86328
E_1	0.00134505	0.0330485	0.1281622	0.4849151	2.769136	10.48978	24.09509	43.24231
E_2	0.001344893	0.03273785	0.1224177	0.4226416	2.189147	8.30195	18.47993	32.73484
E_3	0.001346582	0.03366436	0.1346554	0.5385994	3.365998	13.46364	30.29302	53.85415
E_4	0.001344888	0.03267327	0.1186802	0.3481393	1.683738	5.554621	12.93236	22.63588
E_5	0.001344906	0.03274865	0.1220006	0.3995832	1.6902	6.449636	13.80009	25.12668
E_6	0.00134505	0.03304979	0.1281983	0.4855313	2.781283	10.5562	23.23618	40.81639
E_7	0.001345046	0.03324542	0.1302694	0.4523458	2.295357	10.94049	25.53724	46.09026
E_8	0.001344794	0.03240041	0.1119576	0.2376889	0.7647709	5.331445	13.55212	25.38186
E_9	0.001345027	0.03331753	0.1327083	0.5298099	3.309715	13.23913	29.78856	52.95801
E_{new}	***0.000696124***	***0.001170942***	***0.0017822***	***0.0116919***	***0.1158654***	***0.3593233***	***0.5407395***	***0.665192***

Note that: bold and italic values indicate the smallest values of the AMSE.